

Mathematical Proof as Disciplinary Discourse: Verbal-Symbolic Meaning-Making in Graduate Proof Writing

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Abstract

This article examines how natural language and mathematical notation organize reasoning in twenty graduate-level proof texts from one university in the southern Philippines. A qualitative corpus-based analysis combines Systemic Functional Linguistics, speech-act perspectives, and content analysis to examine verbal-symbolic coordination, recurrent linguistic features, and their potential contribution to inferential transparency. Language introduced objects and assumptions, marked warrants and consequences, maintained reference, and signaled conclusions. Recurrent resources included discourse markers, conditionals, directives, assertions, impersonal formulations, and technical vocabulary. The study contributes a proof-level functional account linking these resources to the status and progression of mathematical claims in student-produced texts. Close analysis also shows that explicit transitions and fluent explanation can coexist with incomplete or circular justification. Persuasiveness is interpreted as textual support for reconstructing and evaluating reasoning; reader comprehension and persuasion were not measured. The findings are limited to the selected corpus and suggest that proof instruction can use annotation and revision to connect linguistic choices with mathematical justification.

1. INTRODUCTION

Mathematical proof is distinguished from other forms of justification by the requirement that a conclusion be warranted by accepted premises, definitions, axioms, or previously established results through valid reasoning. Yet a proof is rarely encountered as a bare formal derivation. It is normally written for a reader and therefore realized through a combination of mathematical notation and natural language. A statement such as $\forall x \in A, x \in B$ is compact, but the argumentative work of explaining why the inclusion $A \subseteq B$ follows often depends on verbal framing, definitions, and inferential markers. This communicative dimension is consistent with

A. J. Stylianides' (2007) characterization of proof as a mathematical argument whose accepted statements, modes of reasoning, and forms of expression must be appropriate to the relevant mathematical community. It also accords with Sundstrom's (2020) treatment of proof writing as a practice integrating logical reasoning, formal methods, and mathematical communication. More broadly, research on proof and proving treats proof as both a disciplinary practice and a central object of teaching and learning (Hanna & de Villiers, 2012; Harel & Sowder, 1998, 2007; Mariotti, 2006; G. J. Stylianides et al., 2024).

Research on undergraduate proof writing has shown that linguistic convention is not peripheral to mathematical competence. Lew and Mejía-Ramos (2019) found that mathematicians attend to conventions governing object introduction, academic language, and contextual formality, whereas students may not fully recognize that these conventions are part of legitimate proof writing. Their subsequent work further demonstrated that judgments about appropriate proof language vary across pedagogical contexts such as textbooks, classroom board work, and student submissions (Lew & Mejía-Ramos, 2020). These findings shift the problem from a simple opposition between "words" and "symbols" toward a functional question: how are verbal and symbolic resources coordinated so that a proof is mathematically acceptable and communicatively transparent? Difficulties with proof writing and comprehension documented elsewhere likewise show that mathematical correctness and communicative accessibility cannot be treated as wholly separate concerns (Azrou & Khelladi, 2019; Mejía-Ramos et al., 2012, 2017; Moore, 1994; Weber, 2015).

This study is positioned within a language-and-mathematics tradition that treats mathematical learning as participation in a disciplinary register realized through verbal, symbolic, and other semiotic resources (Wilkinson, 2019). Recent research reinforces two points central here: disciplinary language is patterned by subject-specific linguistic demands (Cruz Neri et al., 2025), and close discourse analysis can reveal how linguistic choices organize mathematical reasoning and participation (Zhang et al., 2025). Research on mathematics classrooms also shows that mathematical meaning is mediated through coordinated linguistic and multimodal resources rather than through a single representational channel (Tai, 2023). The present article extends this line of inquiry from classroom interaction and curricular texts to student-produced written proofs.

A growing body of scholarship also challenges the view that proofs consist only of assertions. Tanswell and Inglis (2024), using philosophical corpus linguistics, documented the prevalence of imperatives and instructional verbs in research proofs and proposed a "recipe" model in which proof discourse can direct readers to carry out mathematical actions. Contemporary work in the philosophy of mathematical language similarly treats directives and assertions as structurally significant in mathematical argumentation (Caponetto et al., 2023; Ruffino et al., 2021). Thus, expressions such as "Let $x \in A$," "Assume P ," "Choose $\varepsilon > 0$," and "Suppose, for contradiction, that $\neg Q$ " are not merely stylistic openings; they establish objects, conditions, and inferential commitments on which subsequent symbolic reasoning depends (Kontorovich et al., 2024; Tanswell & Inglis, 2024).

Systemic Functional Linguistics (SFL) provides a complementary basis for explaining why these linguistic choices matter. Halliday and Matthiessen (2014) conceptualize language as a meaning-making system simultaneously serving ideational, interpersonal, and textual

functions. In a proof, the ideational dimension represents mathematical relationships, the textual dimension organizes clauses and expressions into a coherent argument, and the interpersonal dimension contributes to stance and epistemic force. Hence the movement from “ $x \in A$ ” to “therefore $x \in B$ ” is not simply a sequence of tokens: it is a linguistically marked relation between a mathematical condition and a warranted consequence (Morgan, 2006; Planas & Pimm, 2024). Recent SFL scholarship likewise characterizes language as a meaning-making resource, emphasizing that linguistic choices realize multiple modes of meaning in educational discourse (Matthiessen & Yousefi, 2022). In mathematics education, Planas et al. (2023) similarly draw on SFL to describe mathematical teaching talk as participating in layered processes of meaning-making.

Previous studies have established the importance of proof-writing conventions, comprehension, and the coordination of mathematical representations (Lew & Mejía-Ramos, 2019, 2020; Lockwood et al., 2020; Mejía-Ramos et al., 2012). The present study builds on these accounts by examining how linguistic forms and notation jointly organize the progression of complete student-produced proof texts. Its contribution is a corpus-specific functional account connecting object introduction, assumption management, warrants, reference tracking, and conclusion signaling within twenty graduate-level texts. This account brings SFL and speech-act analysis together to distinguish the discourse function of a proof step from the adequacy of its mathematical justification. The contribution lies in this integrated application to the selected texts, rather than in proposing verbal-symbolic hybridity as a previously unrecognized property of proof.

In this study, mathematical action refers to the work performed by linguistic expressions in constructing and advancing a proof—for example, introducing mathematical objects, establishing assumptions, specifying conditions, invoking warrants, directing inferential moves, and asserting conclusions. Language is therefore treated not merely as a means of describing mathematical reasoning but as one of the resources through which that reasoning is enacted in written proof.

Accordingly, the study addresses three research questions: (1) What roles do natural language and mathematical notation perform in the selected proof texts? (2) Which linguistic features recur in these texts, and what mathematical or discourse functions do they serve? (3) How can these features support or limit the textual reconstruction and evaluation of the reasoning? Mathematical persuasiveness is examined through inferential transparency, not through measured reader response. Linguistic fluency cannot establish validity; the analysis concerns how assumptions, warrants, and conclusions are presented for evaluation (Durand-Guerrier et al., 2012; Weber, 2015).

2. THEORETICAL FRAMEWORK

2.1. Proof as a Verbal-Symbolic Mathematical Text

Proof writing moves between symbolic compression and linguistic expansion. Mathematical symbols such as $\in$, $\subseteq$, $\cup$, $\cap$, $=$, $\neq$, $\Rightarrow$, $\Leftrightarrow$, $\forall$, $\exists$, and $\neg$ encode relations with high precision. For example, “ $x \in A \cap B$ ” compresses the proposition that x is simultaneously an element of A and an element of B , while “ $A \subseteq B$ ” expresses universal containment: $\forall x (x \in A \Rightarrow x \in B)$. Symbolic notation therefore carries dense mathematical content. However, the progression from one expression to another generally requires the writer to signal whether the relation is definitional, inferential, conditional, contradictory, or conclusive. Sundstrom (2020) treats

mathematical writing as integral to learning proof, while Lew and Mejía-Ramos (2019, 2020) show that conventional language choices are part of what mathematicians recognize as acceptable proof discourse. This verbal-symbolic coordination is also consistent with broader semiotic accounts of mathematical meaning-making (Ernest, 2006; Godino et al., 2011; Planas & Pimm, 2024; Sáenz-Ludlow & Kadunz, 2016). This use of meaning-making is also consistent with recent multimodal mathematics research, which treats learners' coordination and interpretation of communicative resources as part of mathematical meaning-making (Norberg, 2023).

The relationship is therefore complementary rather than competitive. Symbols can reduce redundancy but also increase interpretive density; natural language can explicate inferential dependencies but may become verbose if it merely restates notation. An effective proof coordinates both. For instance, “Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. Since $A \subseteq C$, it follows that $x \in C$ ” uses notation for membership and inclusion while assigning language the task of introducing an arbitrary element, unpacking a definition, and marking the implication. Such coordination is consistent with research showing that proof comprehension depends on recognizing the status, structure, and function of successive proof components (Lockwood et al., 2020; Mejía-Ramos et al., 2012; Weber, 2015).

2.2. Systemic Functional Linguistics and Proof Organization

SFL is particularly useful for a study of proof language because it interprets grammatical and lexical choices in relation to what they accomplish in discourse (Halliday & Matthiessen, 2014). The ideational function is visible when clauses encode relations such as implication, membership, equality, convergence, divisibility, or contradiction. Conditional constructions such as “if P , then Q ” closely mirror the logical relation $P \Rightarrow Q$. Existential language such as “there exists $n \in \mathbb{N}$ ” corresponds to $\exists n \in \mathbb{N}$, while universal expressions such as “for every $\varepsilon > 0$ ” realize the quantificational structure $\forall \varepsilon > 0$ (Morgan, 2006; Planas & Pimm, 2024).

The textual function concerns coherence and cohesion. Proof markers such as “since,” “then,” “therefore,” “thus,” “hence,” “consequently,” and “similarly” orient the reader toward the relationship between adjacent propositions. Referential expressions such as “this,” “that,” and “such” can link a current claim to a previously established step, while repeated mathematical terms preserve lexical continuity. In SFL terms, these devices help transform a series of locally correct statements into a text whose global inferential structure can be reconstructed (Halliday & Matthiessen, 2014; Morgan, 2006). Broader research on mathematical language likewise identifies text-level linguistic organization as relevant to mathematics comprehension, although it does not by itself establish the specific functions of these forms in proof (Cruz Neri & Retelsdorf, 2022).

The interpersonal dimension concerns how claims are positioned in the text. Formulations such as “it follows that” or “it is shown that” can foreground an intended consequence while minimizing explicit reference to the writer. They may contribute to an impersonal disciplinary stance, but neither impersonality nor an assertive tone establishes objectivity, authority, or mathematical validity. These interpretations concern textual positioning rather than demonstrated effects on readers (Halliday & Matthiessen, 2014; Lew & Mejía-Ramos, 2019; Planas & Pimm, 2024).

2.3. Speech Acts, Directives, and Mathematical Action

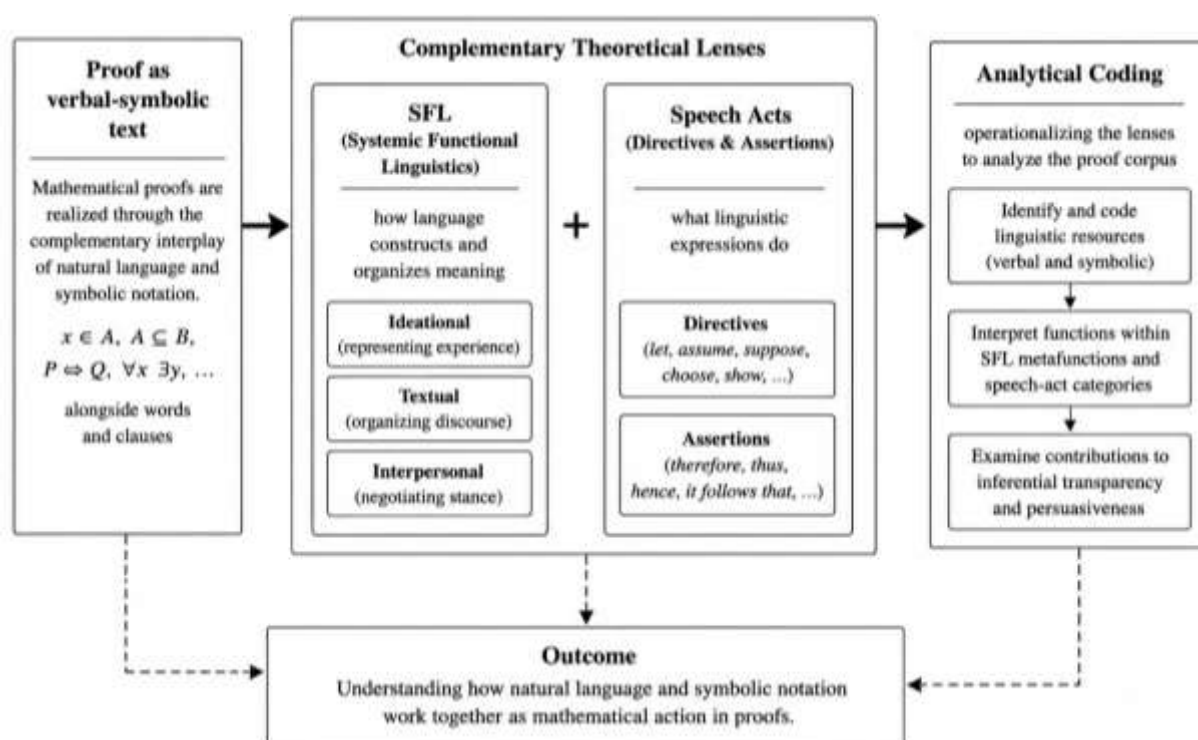
Proof discourse also performs actions. “Let $x \in A$,” “assume P ,” “choose $n \in \mathbb{N}$,” and “suppose $\neg Q$ ” establish conditions for subsequent reasoning; “therefore Q ” and “hence $A = B$ ” assert conclusions. Ruffino et al. (2021) analyze distinctive speech acts in mathematics, while Caponetto et al. (2023) show how directives can participate in supporting mathematical assertions. Tanswell and Inglis (2024) provide corpus evidence that imperatives occur disproportionately within proofs, reinforcing the view that readers are often instructed to consider, choose, define, assume, or construct mathematical objects. These sources provide direct support for treating directives and assertions as functional components of proof discourse (Caponetto et al., 2023; Ruffino et al., 2021; Tanswell & Inglis, 2024).

This perspective is especially relevant to graduate student proofs. A directive such as “Let $x \in A$ ” reduces ambiguity by fixing an arbitrary element, while “Assume, for contradiction, that $\sqrt{2} \in \mathbb{Q}$ ” establishes a temporary proposition to be discharged. The linguistic act structures the proof’s local logical state. Directive and assertive forms are therefore analyzed here not as decorative grammar but as functional elements in the progression from hypothesis to conclusion (Caponetto et al., 2023; Tanswell & Inglis, 2024).

2.4. Integrated Theoretical Framework

Taken together, these perspectives conceptualize mathematical proof as a disciplinary text in which verbal and symbolic resources jointly realize mathematical reasoning. The verbal-symbolic perspective establishes that notation and natural language distribute meaning-making work across the proof; Systemic Functional Linguistics provides a functional account of how linguistic choices construe mathematical relations, organize inferential progression, and position claims within disciplinary discourse; and speech-act theory explains how directives and assertions perform mathematical actions such as introducing objects, establishing assumptions, and advancing conclusions.

Figure 1. *The Integrated Theoretical Framework*



These perspectives are complementary rather than competing because they operate at different analytical levels: the first specifies the semiotic relationship under investigation, SFL explains how language realizes disciplinary meaning, and speech-act theory clarifies what particular utterances do within the unfolding argument. Accordingly, meaning-making is used here in its established functional and semiotic sense: the production and interpretation of disciplinary meaning through available linguistic, symbolic, and other semiotic resources (Matthiessen & Yousefi, 2022; Norberg, 2023; Planas et al., 2023).

The framework positions verbal-symbolic coordination as the central object of inquiry, with SFL and speech-act theory providing complementary theoretical lenses for examining mathematical proof. SFL explains how linguistic resources construe and organize disciplinary meaning, whereas speech-act theory accounts for how particular expressions perform mathematical actions within an argument. Together, these perspectives guide the analytical coding and interpretation of verbal and symbolic resources, their linguistic and mathematical functions, and their contributions to inferential transparency and persuasiveness. Figure 1 illustrates these relationships and their operationalization in the empirical analysis of the twenty-proof corpus (Halliday & Matthiessen, 2014; Planas & Pimm, 2024).

The framework guides three analytical dimensions: the functions of verbal and symbolic resources, the recurrent linguistic forms realizing those functions, and the visibility of assumptions, warrants, transitions, and conclusions. In Figure 1, expressions such as “therefore” and “hence” are markers accompanying assertions, rather than assertions by themselves. Persuasiveness denotes the analyst’s interpretation of the text’s support for reconstructing and evaluating reasoning. It does not denote a measured reader effect or certify correctness (Lockwood et al., 2020; Mejía-Ramos et al., 2012; Weber, 2015).

3. METHODOLOGY

3.1. Research Design and Corpus

The study used a qualitative corpus-based design integrating corpus-oriented linguistic analysis, SFL-informed functional interpretation, speech-act analysis, and content analysis. Corpus approaches are well suited to identifying recurrent forms and their distribution in authentic texts (Biber et al., 1998; McEnery & Hardie, 2012). For proof discourse specifically, Tanswell and Inglis (2024) demonstrate the value of corpus methods for examining instructions and imperatives, while Lew and Mejía-Ramos (2019, 2020) show why proof writing should be treated as a recognizable disciplinary practice with context-sensitive linguistic conventions.

The corpus consisted of twenty graduate-level student proof texts produced in proof-intensive coursework at a private university in the southern Philippines during academic years 2022–2023 to 2024–2025. The source texts included classwork, homework, quiz, and examination proofs from areas such as set theory, number theory, probability, advanced calculus, linear algebra, and complex analysis. The corpus included formal and relatively informal proofs but was restricted to texts written primarily in English.

The twenty proofs were selected purposively because they contained substantial proof construction rather than isolated calculations. Their mathematical content included relations such as set membership ($x \in A$), subset inclusion ($A \subseteq B$), equality of sets ($A = B$), logical implication ($P \Rightarrow Q$), biconditional relations ($P \Leftrightarrow Q$), negation ($\neg P$), quantification ($\forall, \exists$), algebraic identities, rational and irrational numbers, convergence, and countability. This

mathematical diversity was useful because it allowed linguistic patterns to be examined across different symbolic environments. The purposive, information-oriented logic of this selection is consistent with qualitative sampling principles (Patton, 2015).

The sampling strategy was purposive and information-oriented rather than statistically representative. The analytic goal was to obtain a manageable corpus with sufficient variation in mathematical domain, symbolic density, and discourse organization for close comparison across texts. Accordingly, claims in this article concern recurrent patterns within the analyzed corpus and are not presented as population estimates.

3.2. Analytical Framework and Procedures

The theoretical perspectives outlined above were operationalized through three interrelated analytical dimensions: verbal-symbolic coordination, linguistic features and their mathematical functions, and persuasive or interpretive contribution. Table 1 summarizes the evidence examined, the corresponding coding focus, and the interpretive purpose of each dimension. The dimensions were applied across the full twenty-proof corpus and then examined in greater depth through the focal proofs selected for close discourse analysis.

For verbal-symbolic coordination, each text was examined for how clauses and mathematical expressions introduced objects, expressed relations, and connected proof steps. The analysis did not calculate word–symbol ratios or standardized frequency estimates. A quantifier such as $\forall$ can express the content of a verbal phrase, while a short connective can mark an inferential relation; token counts alone would therefore answer a different question from the functional one pursued here (Biber et al., 1998; McEnery & Hardie, 2012).

Table 1. *Analytical framework for examining language and symbolic notation in mathematical proofs*

Analytical dimension	Primary evidence	Coding focus	Interpretive purpose
Natural language and symbolic notation	Verbal clauses, mathematical expressions, mixed units	Functional contribution of natural language and notation	Describe how verbal and symbolic resources organize proof steps
Linguistic features of proof discourse	Words, clauses, transitions, referential expressions	Discourse markers; conditionals; directives/assertives; cohesion; passive/impersonal forms; technical vocabulary	Identify recurrent linguistic resources in proof discourse
Persuasive and interpretive functions of language	Relationship between linguistic form and proof progression	Explicit warrants; assumptions; reference; claim positioning; conclusion signaling	Interpret textual support for reconstructing and evaluating reasoning

The second dimension examined recurrent linguistic features at lexical, clausal, and discourse levels. Coding attended to inferential and causal markers such as “therefore,” “thus,” “hence,” “since,” and “because”; conditional constructions such as “if ... then ...”; directive forms such

as “let,” “assume,” “suppose,” “choose,” and “show”; assertive and declarative statements; referential resources such as “this,” “that,” and “such”; passive and impersonal constructions such as “it follows that” and “it is shown that”; and discipline-specific vocabulary such as “subset,” “divisible,” “convergent,” “Cauchy sequence,” and “countable.” These forms were interpreted according to the mathematical actions and discourse functions they realized—for example, introducing an object, establishing a condition, identifying a warrant, maintaining reference, signaling an inference, or asserting a conclusion. This functional orientation draws most directly on SFL and speech-act accounts of language as meaning-making and action (Caponetto et al., 2023; Halliday & Matthiessen, 2014; Ruffino et al., 2021; Tanswell & Inglis, 2024).

The third dimension examined whether the presentation made assumptions, definitions, warrants, referents, and conclusions identifiable. Persuasive or interpretive contribution was inferred from these textual relations, without reader testing or expert clarity ratings. A transition marker was analyzed separately from the warrant for the inference it introduced: an explicit “therefore” could mark an intended consequence even where the preceding justification was incomplete (Lockwood et al., 2020; Mejía-Ramos et al., 2012, 2017; Weber, 2015).

3.3. Analytical Rigor and Ethical Considerations

Each proof text was annotated at lexical, clausal, and whole-text levels for verbal-symbolic coordination, discourse markers, conditionals, directives and assertions, reference, technical vocabulary, and argumentative function. The same categories were applied across the twenty texts, and the resulting interpretations were compared for recurrent patterns and contrasting cases. Corpus-oriented analysis identified forms, SFL informed their textual and interpersonal interpretation, and speech-act analysis distinguished introductions and assumptions from assertions. These complementary perspectives supported interpretation but did not provide independent validation of the findings. No formal inter-coder agreement coefficient was calculated.

The proofs were anonymized, and identifying institutional information is withheld to comply with double-anonymized review. The study treated the student texts as educational documents rather than as measures of individual linguistic deficiency. This distinction is important because proof writing conventions are learned disciplinary practices, and students’ deviations from expert norms may reflect limited exposure to those conventions rather than lack of mathematical capability (Lew & Mejía-Ramos, 2019, 2020).

3.4. Selection of Focal Proofs for Close Discourse Analysis

Figure 2. Proof 3. Element-wise set reasoning: $A \subseteq B$ implies $A - C \subseteq B - C$.

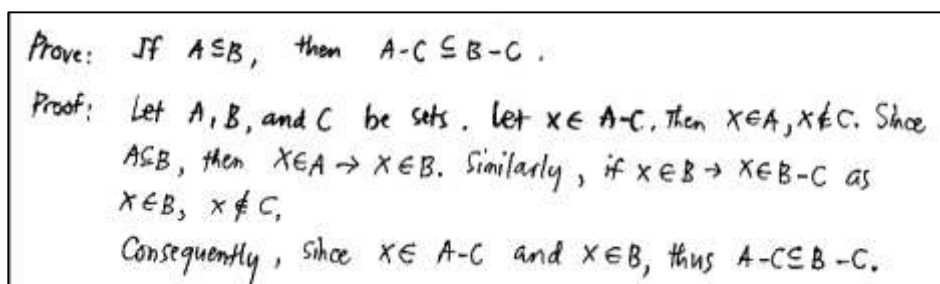


Figure 3. Proof 12. Cauchy-sequence argument using ε , N , and convergence to L .

Prove that if a_n is a Cauchy sequence, then a_n converges. <u>Proof:</u> <u>Step 1:</u> Assume a_n is a Cauchy sequence. For every $\varepsilon > 0$, there exists $N > 0$ such that for all $m, n \geq N$, $ a_n - a_m < \varepsilon.$ <u>Step 2:</u> Use the completeness of $\mathbb{R}$. The real numbers $\mathbb{R}$ are complete, which means that every Cauchy sequence in $\mathbb{R}$ has a limit $L \in \mathbb{R}$. The terms of a Cauchy sequence get arbitrarily close to each other, so they must "cluster" around a specific point L.	<u>Step 3:</u> Show $a_n \rightarrow L$. $ a_n - a_m < \frac{\varepsilon}{2}.$ Fix $n \geq N$. Since a_n is a Cauchy sequence, a_n converges to L. Thus, for large enough m, I choose m such that $a_n - a_m < \frac{\varepsilon}{2}$ $ a_m - L < \frac{\varepsilon}{2}.$ Use triangle inequality: $ a_n - L \leq a_n - a_m + a_m - L .$ For $m, n \geq N$, $a_n - a_m < \frac{\varepsilon}{2}$ and $a_m - L < \frac{\varepsilon}{2}$. Hence, $ a_n - L < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$ <u>Step 4:</u> Conclude the proof. Since $a_n - L < \varepsilon$ for all $\varepsilon > 0$, the sequence a_n converges to L. <u>Conclusion:</u> It is proven that every Cauchy sequence in $\mathbb{R}$ converges to a limit $L \in \mathbb{R}$, completing the proof.
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Figure 4. Proof 16. Cantor diagonalization argument for the uncountability of the real numbers.

Show that the set of real numbers is uncountable.

The Proof:

1. Assume the contrary:
Suppose the real numbers in the interval $[0, 1]$ are countable.
Then, we could list all these real numbers as a sequence:
$$r_1, r_2, r_3, \dots$$

where each r_i is written in its decimal expansion:

$$r_1 = 0.a_{11}a_{12}a_{13}\dots, r_2 = 0.a_{21}a_{22}a_{23}\dots, r_3 = 0.a_{31}a_{32}a_{33}\dots, \text{ and so on.}$$

Here, a_{ij} denotes the j -th digit of the i -th number in the list.
2. Construct a new number.
Now, construct a new real number x in $[0, 1]$ such that x differs from every r_i in at least one decimal place.
Define the decimal expansion of x as follows:

$$x = 0.b_1b_2b_3\dots$$

where:

$$b_i = \begin{cases} 1 & \text{if } a_{ii} \neq 1, \\ 2 & \text{if } a_{ii} = 1. \end{cases}$$

This ensures that x differs from r_1 in the first digit ($b_1 \neq a_{11}$), r_2 in the second digit ($b_2 \neq a_{22}$), r_3 in the third digit ($b_3 \neq a_{33}$), and so on.
3. x is not in the list:
By construction, x differs from every r_i in at least one decimal place. Therefore, x cannot be equal to any r_i , meaning x is not in the list.
4. Contradiction: This contradicts the assumption that all real numbers in $[0, 1]$ can be listed. Hence, the set of real numbers in $[0, 1]$ is uncountable.

Extension to All Real Numbers
The real numbers $\mathbb{R}$ are uncountable because the subset $[0, 1]$ is uncountable, and a subset of an uncountable set must also be uncountable.

Conclusion: The set of real numbers is uncountable.

All twenty texts were included in the corpus-level analysis. Three information-rich cases are reproduced for closer examination: an element-wise set argument (Corpus Proof 3), a Cauchy-sequence argument (Corpus Proof 12), and a Cantor diagonalization argument (Corpus Proof 16). These cases offer contrasting mathematical contexts and make the distinction between discourse organization and mathematical justification visible.

4. RESULTS AND DISCUSSION

Figures 2–4 provide focal cases within the twenty-text analysis. The discussion distinguishes observed wording in these reproduced texts from schematic examples used to explain the analytical categories. The latter illustrate mathematical and discourse functions and are not presented as verbatim corpus quotations or frequency evidence.

4.1. Functions of Natural Language and Symbolic Notation

In the selected corpus, natural language accompanied notation in introducing objects, specifying assumptions, invoking definitions, and marking the intended progression toward a conclusion. The analysis identifies these functions rather than a numerical predominance of words over symbols. This finding is consistent with established accounts of proof-writing conventions and mathematical language (Lew & Mejía-Ramos, 2019, 2020; Planas & Pimm, 2024).

Corpus Proof 3 (Figure 2) begins “Let A , B , and C be sets” and “Let $x \in A - C$.” The second directive introduces the element used in the inclusion argument. The following statement records $x \in A$ and $x \notin C$, while “Since $A \subseteq B$ ” invokes the hypothesis licensing $x \in B$. These moves make the intended route to $x \in B - C$ reconstructible. However, the use of “Similarly” compresses the final membership step, and the concluding sentence does not explicitly repeat $x \notin C$ or the arbitrariness of x . The argument can be reconstructed from the preceding statements, but the presence of transition markers alone does not make every dependency explicit.

Corpus Proof 12 (Figure 3) uses numbered steps, “For every $\varepsilon > 0$,” “there exists N ,” and “Use triangle inequality” to organize an intended convergence argument. These expressions identify quantifier scope and the proposed estimate. The text nevertheless states that completeness means every Cauchy sequence has a limit and then uses “Since a_n is a Cauchy sequence, a_n converges to L ” while purporting to establish that conclusion. If this characterization of completeness is an accepted premise, the convergence claim follows from that premise; if convergence of Cauchy sequences is the theorem to be proved from another completeness axiom, the text does not independently establish it. The subsequent choice of m already presupposes convergence to L . This case shows how explicit linguistic organization can coexist with an unresolved justificatory dependency.

Corpus Proof 16 (Figure 4) separates a temporary assumption from the conclusion through “Suppose the real numbers in the interval $[0,1]$ are countable” and the later heading “Contradiction.” The directives “construct a new number” and “Define the decimal expansion” introduce x and the digits b_i . Choosing b_i as 1 when $a_{ii} \neq 1$ and as 2 otherwise ensures a difference at the diagonal position; the restriction to digits 1 and 2 also avoids a terminating or recurring-9 expansion for x . The verbal statement that x is absent from the proposed list then links the construction to the contradiction. The representation of every listed element as $0.a_{i1}a_{i2}\dots$ leaves the endpoint 1 and decimal-representation conventions implicit, so a fully

explicit version would handle those details. These features distinguish the organization of the contradiction from the completeness of its written formulation.

Table 2 summarizes schematic verbal-symbolic pairings used to interpret these functions. The examples illustrate the coding categories; they are not a count of observed forms or a verbatim transcription of the focal texts.

Table 2. *Schematic coordination of natural language and mathematical symbolism*

Proof function	Symbolic form	Typical verbal realization	Why the combination matters
Introduce an arbitrary object	$x \in A$	Let $x \in A$.	Introduces x ; arbitrariness must be established by its role and the surrounding argument.
Unpack a definition	$x \in A \cap B$ $\Rightarrow x \in A \wedge x \in B$	Then $x \in A$ and $x \in B$.	Makes the definitional relation explicit.
Express inclusion	$A \subseteq B$	For every x , if $x \in A$, then $x \in B$.	Connects compact notation to its quantified meaning.
State implication	$P \Rightarrow Q$	Assume P . If P , then Q . Therefore Q .	Distinguishes the implication from the assertion of Q under the premise P .
State equivalence	$P \Leftrightarrow Q$	We prove both directions.	Signals that $P \Rightarrow Q$ and $Q \Rightarrow P$ are both required.
Contradiction	$\neg P \Rightarrow \perp$	Assume $\neg P$. This contradicts ...	Distinguishes a temporary assumption from the final claim.
Convergence	$\forall \varepsilon > 0, \exists N \in \mathbb{N},$ $\forall n \in \mathbb{N}$ $(n \geq N \Rightarrow a_n - L < \varepsilon)$	Let $\varepsilon > 0$. Choose $N \dots$ For $n \geq N \dots$	Clarifies quantifier order and dependencies.

4.2. Recurrent Linguistic Resources and Their Functions

The analysis identified discourse markers, conditionals, directives, assertions, referential expressions, impersonal formulations, and technical terms in the selected texts. Table 3 organizes these forms by their proposed functions. These are qualitative categories, not ranked frequencies or evidence of a stable repertoire across graduate mathematics generally. SFL supports interpreting the relationship between form and discourse function (Halliday & Matthiessen, 2014), while research on disciplinary texts provides a broader context for subject-specific linguistic patterns (Cruz Neri et al., 2025).

Markers such as “since,” “then,” “thus,” and “hence” indicated intended relations among propositions. In Proof 3, “Since $A \subseteq B$ ” points to the premise used for membership in B ; in Proof 16, “Therefore” links the diagonal construction to the claim that x is absent from the list. Such markers label inferential relations, but their appropriateness depends on the reasoning they connect (Halliday & Matthiessen, 2014; Morgan, 2006).

Conditional constructions organize hypothesis–conclusion relations. Schematically, “if P, then Q” expresses $P \Rightarrow Q$, while “conversely” can signal the second obligation in a biconditional argument. These functions concern implication and scope rather than a general increase in persuasive force (Durand-Guerrier et al., 2012; Mariotti et al., 2018).

Directives such as “let,” “assume,” “suppose,” “choose,” and “consider” introduced objects, local assumptions, or proposed mathematical actions. Their use in the selected texts is consistent with speech-act analyses of mathematical language and the instructional forms documented in professional proofs (Caponetto et al., 2023; Tanswell & Inglis, 2024). An instruction to choose an object does not, by itself, justify that object’s existence.

Table 3. *Recurrent linguistic features and their mathematical-discourse functions*

Feature	Representative forms	Mathematical function	Potential discourse function
Discourse markers	since, then, therefore, thus, hence, similarly	Mark warrants, consequences, analogy, closure	Can make intended inferential relations visible
Conditionals	if...then; whenever; provided that	Realize $P \Rightarrow Q$ and hypothesis–conclusion structure	Can clarify dependency and direction
Directives	let, assume, suppose, choose, consider, show	Establish objects, assumptions, witnesses, tasks	Introduces or directs mathematical action; existence requires justification
Assertives/declaratives	x is irrational; $A \subseteq B$; the sequence converges	State mathematical claims	Projects commitment to propositions
Referential cohesion	this, that, such; repeated x, A, a_n	Maintain object and proposition tracking	Can maintain reference when antecedents are unambiguous
Impersonal/passive forms	it follows that; it is shown that	Foreground consequence over writer	Can function as an impersonal disciplinary stance; does not establish validity
Technical vocabulary	subset, divisible, Cauchy, countable, convergence	Encode domain-specific definitions	Can support precision when definitions are used correctly

Assertive clauses stated propositions and intended consequences. In Proof 16, the statement that x is not in the list follows the construction of its digits; in Proof 12, the assertion of convergence precedes an estimate that depends on that convergence. This contrast makes the status and warrant of an assertion analytically distinct from its declarative form (Caponetto et al., 2023; Ruffino et al., 2021).

Repeated variables and referential expressions linked successive steps. Stable reference to x and the sets in Proof 3 helped preserve the element-wise structure. In Proof 12, ϵ , N, m, n, and L required distinct roles and dependencies, which explicit naming alone did not fully justify. Reference tracking therefore concerns both continuity and the logical status of the named objects (Halliday & Matthiessen, 2014; Lew & Mejía-Ramos, 2019).

Impersonal and passive formulations, including “it follows that” and “it is proven that,” presented claims as mathematical consequences. They can function as conventional markers of stance, but a closing assertion of proof does not establish that all obligations have been met. Proof 12 provides a case in which conclusive wording accompanies the unresolved dependency identified above (Halliday & Matthiessen, 2014; Morgan, 2006).

Technical terms invoked definitions and accepted mathematical properties. “Subset,” “Cauchy sequence,” and “countable,” for example, carry obligations concerning arbitrary elements, quantified estimates, and enumeration. Their use may contribute to precision when those obligations are respected; terminology alone cannot supply a missing warrant (Lew & Mejía-Ramos, 2019, 2020; Planas & Pimm, 2024).

4.3. Inferential Transparency and Its Limits

The third dimension concerns how the text presents reasoning for evaluation. Table 4 summarizes potential interpretive contributions of the linguistic resources, rather than observed changes in comprehension or persuasion. The focal cases show that visible assumptions and transitions can support reconstruction while also exposing gaps in justification (Lockwood et al., 2020; Mejía-Ramos et al., 2012, 2017; Weber, 2015).

Table 4. *Potential interpretive contributions of linguistic features*

Interpretive dimension	Linguistic resources	Mathematical example	Potential textual contribution
Inferential transparency	since, therefore, hence, it follows that	$x \in A$ and $A \subseteq B$ $\Rightarrow x \in B$	Can identify the intended warrant–conclusion relation
Assumption control	assume, suppose, let	Assume $\neg P$; derive $\perp$	Can distinguish temporary assumptions from endorsed propositions
Logical dependency	if...then; for every; there exists	$\forall \varepsilon > 0, \exists N \in \mathbb{N} \dots$	Can make quantifier order and conditional scope traceable
Cohesion	this, such, repeated variables/terms	$x \rightarrow x$ throughout an element-wise proof	Can maintain continuity when objects and referents remain stable
Claim positioning and lexical precision	precise terminology; impersonal stance	Cauchy, convergent, subset; it follows that	May signal a conventional disciplinary stance; does not establish authority or rigor
Explanatory explicitness	brief explanations around notation	$A \cap B$ unpacked as $x \in A$ and $x \in B$	Can expose the meaning of notation; reader benefit was not measured

A schematic contrast clarifies this distinction: “ $x \in A$. $A \subseteq B$. $x \in B$ ” leaves the relation implicit, whereas “Since $x \in A$ and $A \subseteq B$, it follows that $x \in B$ ” explicitly connects premises to consequence. The second formulation names a valid warrant relation in this example; the same connective in another context would require its own mathematical justification. This comparison illustrates textual explicitness without demonstrating a measured reader benefit (Mejía-Ramos et al., 2012, 2017; Weber, 2015).

The contrast between Proofs 12 and 16 is particularly informative. Both use headings, directives, and conclusion markers. In Proof 12, an asserted limit is used without an independent derivation unless it is already licensed by the chosen completeness premise. In Proof 16, the constructed digits supply the reason for exclusion from the list, although representation conventions remain implicit. Linguistic organization thus makes an intended path visible; the adequacy of that path still requires mathematical scrutiny. This interpretation separates communicative form from warranted inference.

4.4. Contribution of the Integrated Functional Account

The analysis connects linguistic forms with the status of mathematical statements across complete student-produced texts. Its contribution is the combined use of SFL and speech-act perspectives to trace how assumptions, object introductions, warrants, and conclusions are organized, including cases where explicit discourse structure coexists with incomplete justification. This extends the application of established accounts of proof conventions and directives to the selected graduate corpus (Lew & Mejía-Ramos, 2019, 2020; Tanswell & Inglis, 2024). Verbal-symbolic coordination itself is consistent with existing semiotic accounts of mathematics (Planas & Pimm, 2024; Wilkinson, 2019).

The distinction between linguistic organization and mathematical justification also informs the interpretation of student difficulties. An unclear text may obscure a valid idea, while a fluent text may conceal an unsupported inference. Neither possibility permits a conclusion about the writer's understanding from the written product alone. Studies of the transition to proof provide a wider context for difficulties involving language, definitions, and proof organization (Azrou & Khelladi, 2019; Moore, 1994).

5. IMPLICATIONS FOR GRADUATE MATHEMATICS EDUCATION

The findings have implications for disciplinary language pedagogy in advanced mathematics. The corpus analysis suggests that proof language can be taught as a functional resource for mathematical action rather than as a list of preferred phrases. Students need to understand, for example, how “since” introduces a warrant, “therefore” marks a consequence, “suppose” establishes a local assumption, and “it follows that” signals an inference that must already be justified. Making these functions explicit aligns language instruction with proof logic and with research on disciplinary mathematical discourse and proof-writing conventions (Kontorovich et al., 2024; Lew & Mejía-Ramos, 2019, 2020; Wilkinson, 2019).

Proof annotation and reflective revision can make these functions visible. Students can label object introductions, assumptions, definitions, warrants, inferences, contradictions, and conclusions while identifying the linguistic forms that realize those roles. After drafting, they can also check whether every object is introduced, each implication is licensed, referents remain unambiguous, and conclusion markers accurately represent the argument. Such activities may support metalinguistic awareness without separating language from mathematical reasoning (Tanswell & Inglis, 2024; Sundstrom, 2020).

Translation between symbolic and verbal forms should also be incorporated into advanced coursework. Students can expand $\forall x(x \in A \Rightarrow x \in B)$ into natural language, compress a verbal biconditional into $P \Leftrightarrow Q$, distinguish $\neg(\forall x P(x))$ from $\forall x \neg P(x)$, or narrate the dependencies in an ε - N proof. These exercises foreground the coordination of notation, quantification, and disciplinary meaning-making (Pimm, 1987; Planas & Pimm, 2024).

Assessment should distinguish mathematical validity from communicative quality while recognizing both. Eloquence cannot compensate for an invalid proof, but among mathematically sound arguments, explicit justification, cohesive reference, notation consistency, and inferential clarity are educationally meaningful. Rubrics that make these expectations explicit can help students see communicative competence as part of proof competence rather than as an after-the-fact stylistic refinement (Lew & Mejía-Ramos, 2019; G. J. Stylianides & A. J. Stylianides, 2017; Teledahl et al., 2025).

6. LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

The twenty purposively selected texts come from one university and cannot represent graduate proof writing across institutions, disciplines, or language backgrounds. They support qualitative descriptions of the analyzed corpus, not population estimates or causal claims. Institutional context, task demands, and repeated contributions from individual writers may shape the observed patterns; the available analysis does not separate these influences (Creswell & Poth, 2018; Patton, 2015).

No standardized word–symbol ratio or feature–frequency analysis was conducted. Functional descriptions concern what verbal and symbolic units do in context rather than how often they occur. Future studies could define reproducible counting units alongside functional coding, without assuming that a word and a mathematical symbol carry equivalent semantic weight (Biber et al., 1998; McEnery & Hardie, 2012).

The written products do not establish the cognitive processes through which students developed their arguments. Nor were comprehension, interpretive burden, or persuasive effects tested with readers. Think-aloud studies, stimulated recall, revision analysis, and expert judgments could examine these questions directly. Linguistic form, mathematical validity, and reader response would require distinct measures.

A further limitation is that the study did not calculate a formal inter-coder reliability coefficient. The findings should therefore be understood as theoretically informed qualitative interpretations supported by systematic application of the coding framework, rather than as reliability-estimated categorical measurements. Future studies can strengthen reproducibility by using independently coded subsets and reporting agreement statistics alongside qualitative adjudication of disagreements.

Future corpus studies could compare graduate students with professional mathematicians, investigate disciplinary differences among algebra, analysis, topology, and number theory, and test whether specific linguistic patterns predict expert judgments of clarity. Lew and Mejía-Ramos (2020) show that judgments about acceptable proof language vary across pedagogical contexts, while Tanswell and Inglis (2024) demonstrate the value of corpus evidence for characterizing professional proof language. Broader reviews of argumentation and proof research identify complementary opportunities for extending this work across educational levels and mathematical settings (Kartika et al., 2024; Lessing & Ogbonnaya, 2026; G. J. Stylianides et al., 2024).

7. CONCLUSION

In the twenty proof texts from one university, natural language introduced objects and assumptions, marked intended warrants and consequences, and maintained reference alongside

notation. The study contributes an integrated functional account of these relations using SFL and speech-act perspectives. Close analysis also shows that explicit discourse organization can coexist with incomplete justification, so textual fluency cannot certify validity.

These corpus-specific interpretations do not establish population patterns or measured effects on readers. For instruction, they suggest using annotation and revision to connect linguistic choices with the mathematical obligations of each proof step. Further work with broader corpora, independent coding, and reader evaluation is needed to assess transferability and interpretive effects.

8. DECLARATIONS

8.1. Ethics and Consent

The corpus comprised student-written mathematical proofs originally produced as coursework. Informed consent was obtained before the proofs were included in the research corpus, and only materials falling within the scope of that consent were analyzed or reproduced. Before analysis and dissemination, names and other direct identifiers were removed, each proof was assigned a corpus code, and potentially identifying contextual details were withheld. Access to the original files was limited to the research team, and results are reported at corpus level to reduce the risk of re-identification. The study followed the institution's applicable research-ethics and data-protection requirements.

8.2. Data Availability

The research data consist of student-written mathematical proof texts collected as educational documents. Three anonymized focal proof images are reproduced in this article. The original educational documents are not deposited in a public repository because access is subject to the consent, confidentiality, and privacy conditions under which the materials were collected.

9. REFERENCES

- Azrou, N., & Khelladi, A. (2019). Why do students write poor proof texts? A case study on undergraduates' proof writing. *Educational Studies in Mathematics*, 102(2), 257–274. <https://doi.org/10.1007/s10649-019-09911-9>
- Biber, D., Conrad, S., & Reppen, R. (1998). *Corpus linguistics: Investigating language structure and use*. Cambridge University Press.
- Caponetto, L., San Mauro, L., & Venturi, G. (2023). How to make (mathematical) assertions with directives. *Synthese*, 202, 127. <https://doi.org/10.1007/s11229-023-04360-7>
- Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE.
- Cruz Neri, N., Bernholt, S., & Härtig, H. (2025). Every subject has its own language – patterns of linguistic features of expository texts in German STEM textbooks. *Linguistics and Education*, 86, 101405. <https://doi.org/10.1016/j.linged.2025.101405>
- Cruz Neri, N., & Retelsdorf, J. (2022). The role of linguistic features in science and math comprehension and performance: A systematic review and desiderata for future research. *Educational Research Review*, 36, 100460. <https://doi.org/10.1016/j.edurev.2022.100460>
- Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. S., & Tanguay, D. (2012). Argumentation and proof in the mathematics classroom. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education: The 19th ICMI study* (pp. 349–367). Springer. https://doi.org/10.1007/978-94-007-2129-6_15

- Ernest, P. (2006). A semiotic perspective of mathematical activity: The case of number. *Educational Studies in Mathematics*, 61(1–2), 67–101. <https://doi.org/10.1007/s10649-006-6423-7>
- Godino, J. D., Font, V., Wilhelmi, M. R., & Lurduy, O. (2011). Why is the learning of elementary arithmetic concepts difficult? Semiotic tools for understanding the nature of mathematical objects. *Educational Studies in Mathematics*, 77(2–3), 247–265. <https://doi.org/10.1007/s10649-010-9278-x>
- Halliday, M. A. K., & Matthiessen, C. M. I. M. (2014). *Halliday's introduction to functional grammar* (4th ed.). Routledge.
- Hanna, G., & de Villiers, M. (Eds.). (2012). *Proof and proving in mathematics education: The 19th ICMI study*. Springer. <https://doi.org/10.1007/978-94-007-2129-6>
- Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education III* (pp. 234–283). American Mathematical Society. <https://doi.org/10.1090/cbmath/007/07>
- Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805–842). Information Age Publishing.
- Kartika, H., Warmi, A., Urayama, D., & Suprihatiningsih, S. (2024). Mathematical argumentation in higher education: A systematic literature review. *Journal of University Teaching & Learning Practice*, 21(7). <https://doi.org/10.53761/e0vd5v40>
- Kontorovich, I., Liu, N. Q., & Kang, S.-W. (2024). Transitioning to proof via writing scripts on the rules of a new discourse. *Educational Studies in Mathematics*, 117(1), 143–162. <https://doi.org/10.1007/s10649-024-10324-6>
- Lessing, M.-J., & Ogbonnaya, U. I. (2026). Reasoning and proof in mathematics education: A systematic literature review. *Discover Education*, 5, 31. <https://doi.org/10.1007/s44217-025-01016-1>
- Lew, K., & Mejía-Ramos, J. P. (2019). Linguistic conventions of mathematical proof writing at the undergraduate level: Mathematicians' and students' perspectives. *Journal for Research in Mathematics Education*, 50(2), 121–155. <https://doi.org/10.5951/jresmetheduc.50.2.0121>
- Lew, K., & Mejía-Ramos, J. P. (2020). Linguistic conventions of mathematical proof writing across pedagogical contexts. *Educational Studies in Mathematics*, 103(1), 43–62. <https://doi.org/10.1007/s10649-019-09915-5>
- Lockwood, E., Caughman, J. S., & Weber, K. (2020). An essay on proof, conviction, and explanation: Multiple representation systems in combinatorics. *Educational Studies in Mathematics*, 103(2), 173–189. <https://doi.org/10.1007/s10649-020-09933-8>
- Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 173–204). Sense Publishers. https://doi.org/10.1163/9789087901127_008
- Mariotti, M. A., Durand-Guerrier, V., & Stylianides, G. J. (2018). Argumentation and proof. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 75–89). Routledge.

- Matthiessen, C. M. I. M., & Yousefi, M. (2022). Systemic functional linguistics as a resource for teacher education and writing development: Emerging properties and applications. *Language, Context and Text*, 4(1), 114–145. <https://doi.org/10.1075/langct.20005.mat>
- McEnery, T., & Hardie, A. (2012). *Corpus linguistics: Method, theory and practice*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511981395>
- Mejía-Ramos, J. P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, 79(1), 3–18. <https://doi.org/10.1007/s10649-011-9349-7>
- Mejía-Ramos, J. P., Lew, K., de la Torre, J., & Weber, K. (2017). Developing and validating proof comprehension tests in undergraduate mathematics. *Research in Mathematics Education*, 19(2), 130–146. <https://doi.org/10.1080/14794802.2017.1325776>
- Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*, 27(3), 249–266. <https://doi.org/10.1007/BF01273731>
- Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics*, 61(1–2), 219–245. <https://doi.org/10.1007/s10649-006-5477-x>
- Norberg, M. (2023). Young students’ meaning-making when working with mathematics textbooks – A multimodal study focusing on the designed and the discovered. *Research in Mathematics Education*, 25(2), 194–218. <https://doi.org/10.1080/14794802.2022.2045624>
- Patton, M. Q. (2015). *Qualitative research & evaluation methods: Integrating theory and practice* (4th ed.). SAGE.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge.
- Planas, N., Adler, J., & Mwadzaangati, L. (2023). What is mathematics teaching talk for? A response based on three sites of practice in mathematics education. *ZDM – Mathematics Education*, 55, 521–534. <https://doi.org/10.1007/s11858-022-01452-5>
- Planas, N., & Pimm, D. (2024). Mathematics education research on language and on communication including some distinctions: Where are we now? *ZDM – Mathematics Education*, 56(1), 127–139. <https://doi.org/10.1007/s11858-023-01497-0>
- Ruffino, M., San Mauro, L., & Venturi, G. (2021). Speech acts in mathematics. *Synthese*, 198(10), 10063–10087. <https://doi.org/10.1007/s11229-020-02702-3>
- Sáenz-Ludlow, A., & Kadunz, G. (Eds.). (2016). *Semiotics as a tool for learning mathematics*. Sense Publishers. <https://doi.org/10.1007/978-94-6300-337-7>
- Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, 38(3), 289–321. <https://doi.org/10.2307/30034869>
- Stylianides, G. J., & Stylianides, A. J. (2017). Research-based interventions in the area of proof: The past, the present, and the future. *Educational Studies in Mathematics*, 96(2), 119–127. <https://doi.org/10.1007/s10649-017-9782-3>
- Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: A systematic review. *ZDM – Mathematics Education*, 56(1), 47–59. <https://doi.org/10.1007/s11858-023-01518-y>
- Sundstrom, T. (2020). *Mathematical reasoning: Writing and proof* (Version 3). Grand Valley State University Libraries. <https://scholarworks.gvsu.edu/books/24/>
- Tai, K. W. H. (2023). Creating translanguaging spaces in a Hong Kong English medium instruction mathematics classroom: A comparative analysis of classroom interactions

- with and without the use of iPad. *Linguistics and Education*, 78, 101227. <https://doi.org/10.1016/j.linged.2023.101227>
- Tanswell, F. S., & Inglis, M. (2024). The language of proofs: A philosophical corpus linguistics study of instructions and imperatives in mathematical texts. In B. Sriraman (Ed.), *Handbook of the history and philosophy of mathematical practice* (pp. 2925–2952). Springer. https://doi.org/10.1007/978-3-031-40846-5_50
- Teledahl, A., Kilhamn, C., Ahl, L. M., & Helenius, O. (2025). Defining and measuring quality in students' mathematical writing: A systematic literature review. *Mathematics Education Research Journal*, 37(3), 557–576. <https://doi.org/10.1007/s13394-024-00501-4>
- Weber, K. (2015). Effective proof reading strategies for comprehending mathematical proofs. *International Journal of Research in Undergraduate Mathematics Education*, 1(3), 289–314. <https://doi.org/10.1007/s40753-015-0011-0>
- Wilkinson, L. C. (2019). Learning language and mathematics: A perspective from Linguistics and Education. *Linguistics and Education*, 49, 86–95. <https://doi.org/10.1016/j.linged.2018.03.005>
- Zhang, H., Jung, H., Sutcliffe, K., Zhu, H., & Park, S. (2025). Analyzing classroom discourse in social justice mathematical modeling contexts. *Linguistics and Education*, 88, 101446. <https://doi.org/10.1016/j.linged.2025.101446>